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Correlation of Combustor Rig Sound Power Data and Theoretical Basis of Results

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A total of 202 data points from 15 different combustors tested on four different rigs were used to generate a correlation of sound power output from combustor rig tests. The combustors were from three different U.S. manufacturers. The data span a sound power range of 60 dB and have previously defied a unified correlation. It is shown using a plane wave theory of direct and indirect combustion noise that the correlation has a theoretical basis.

Nomenclature

a = combustor radius or specific acoustic admittance for isentropic waves
A = area or fractional heat release rate fluctuation,

$$\int_V Q' \cos Kx \, dV \Big/ \int_V \bar{Q} \, dV$$

b = specific acoustic admittance for nonisentropic waves
c = speed of sound
c_p = specific heat at constant pressure
C = circumference
D = denominator defined in Eqs. (12)
f = function defined in Eq. (1)
F = fuel/air ratio
H_{ij} = transfer function between quantity i and quantity j
i = $(-1)^{1/2}$
k = wavenumber, $\omega/\bar{c}$
ℓ = combustor length
M = Mach number
n = unit outward normal vector
N_f = number of fuel nozzles
p = pressure
P = acoustic power
Q = heat release rate per unit volume
R = gas constant
s = entropy
S_i = auto spectrum of quantity i
t_0 = sample time in Fourier transform operation
T = temperature
u = axial velocity
u_{ref} = reference velocity
v = velocity vector
x = axial distance
y,z = transverse dimensions
$\langle\ \rangle$ = time average
α_e = defined in Eqs. (12)
β_w = specific acoustic wall admittance
γ = ratio of specific heats
Δ = change across combustor
K = complex wavenumber
ρ = density

$$\sigma = \frac{1}{A_e} \int_A (s'/\Delta\bar{s}) \, dA$$

$$\Sigma = \frac{1}{A_e} \int_A (s'/c_p) \, dA$$

ω = circular frequency

Subscripts

a = acoustic part of fluctuations
av = representative average value
e = exit of combustor
eq = equivalent reflection free
i = inlet to combustor
ref = reference conditions
v = vortical part of fluctuations
ω = Fourier-transformed quantity

Superscripts

$(\hat{\ })$ = cross section area average
$(\bar{\ })$ = steady
$(\ )'$ = fluctuation about mean

I. Introduction

DURING the past decade there has been rather intense research on the question of "core engine," "core," or "excess" noise from aircraft gas turbine units. This is noise which emanates from the core (working part of the airflow which passes through the turbine) of the engine. All noise due to the fan (on a turbofan engine) or the jet exhaust is excluded; moreover, turbine noise, which is identifiable by itself, is excluded. For several reasons, it has been suspected for some time that processes taking place within the combustor are responsible for core noise, but there has been controversy over the exact nature of the processes.

Reference 1 claims correlation of combustor rig and core noise data on the basis of direct combustion noise as the dominant noise source. Direct combustion noise is a dilation of the flow, and consequent acoustic wave production, due to fluctuations in the aggregate heat release rate in the combustor. On the other hand, Cumpsty[2] claims correlation of core noise results based upon the mechanism of entropy noise. Entropy noise is noise produced by hot or cold spots traversing a pressure gradient such as exists in the nozzle guide vanes preceding the turbine. Moreover, combustor rig tests on a single combustor[3] have recently shown that under conditions of such a pressure gradient entropy noise tends to

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dominate direct combustion noise. Calculations in Ref. 1, however, based upon a particular theory of entropy noise, do not support the contention that entropy noise is a strong noise source in engines.

These controversies are compounded by the fact that recent combustor rig and core noise data, correlated without any theoretical basis, for the devices of Ref. 4 do not agree with the correlations of Ref. 1. It has furthermore been noted in Ref. 3 that there may be another noise source which has not been considered or measured as yet. This new source, called vorticity noise, is caused by flow fluctuations, due to the turbulence, entering the choked or nearly choked flow of the nozzle guide vanes.

The program was undertaken to see if 1) the results of Refs. 1 and 4 are, in fact, compatible, 2) a unified correlation could be developed, and 3) a theoretical explanation for the results could be developed. The concentration here is on combustor rigs, not engines.

II. Theory

General Formulation

The general aeroacoustics formalism to be used for the theory of combustor noise is the linearized vorticity-acoustic field approach. This was used in Ref. 3 but some of the details were omitted there and some modifications are necessary to account for vorticity noise. The approach has a particular simplicity (and is, in fact, only valid) for low frequency noise so that the plane wave mode is the only important acoustic mode. The approach is detailed in Ref. 5 for a constant property mean flow, but substantial modifications are necessary in the case of combustors with variable velocity and thermodynamic properties.

The approach consists of letting the perturbation velocity v' and the perturbation pressure p' be split into a vortical part associated with the turbulence, and a dilational part associated with the acoustic wave motion. That is, $v' = v_v' + v_a'$ and $p' = p_v' + p_a'$ where the velocity components have the property that $\nabla \times v_a = \nabla \cdot v_v = 0$. These forms are placed in the inviscid equations of gasdynamics with chemical heat release and then split into a set of 1) steady-state equations and 2) perturbation equations for the vortical and acoustic components. This is done in Ref. 3. However, it is found that such a split is possible only if the condition $\nabla p_a' \times \nabla \bar{\rho} = 0$ is rigorously met. That is, the gradient of the acoustic pressure must be parallel to the gradient of mean density. The only reasonable flow and acoustic situation which will satisfy this requirement is a nearly one-dimensional mean flow upon which is superimposed plane wave acoustic motion. This will be assumed.

The limit of low Mach number will also be assumed, in which case the differential equation for the Fourier transform of the acoustic pressure becomes[3]

$$\nabla^2 p_\omega + k^2 p_\omega = -f = -\frac{\omega i Q_\omega}{\bar{c}^2}(\gamma - 1) \qquad (1)$$

Also assumed in Eq. (1) is that the speed of sound is roughly constant through the combustor (an average value has been used). The subscript a has been dropped for clarity in Eq. (1) but it is understood that the acoustic pressure obeys Eq. (1). Also, to be consistent with the splitting requirement, the transverse derivatives in Eq. (1) must be weak compared with the longitudinal derivative.

The neglect of terms proportional to the Mach number in Eq. (1) is permissible because there are no essential physics omitted. That is, the effect of the flow terms is minor and is primarily felt in the wave propagation speed so that resonant frequencies are only slightly shifted from their no-flow values. This is also the primary effect of variable speed of sound. The same is not true when one considers the boundary conditions to Eq. (1), where flow terms are essential to a proper description of acoustic damping and noise generation, as will be seen. Consequently, more care is necessary in the boundary conditions formulation. It has furthermore been assumed in Eq. (1) that liquid-gas momentum (drag) interactions are unimportant. This has justification in numerous studies of combustion instability,[6] where the drag effect is of the same order of magnitude as the flow terms. It is, however, a damping effect. It will be effectively taken into account in the combustor liner admittance, which also damps acoustic motions.

Cross-section averages of terms will be denoted by a hat superscript. That is, for example,

$$\hat{p}_\omega = \frac{1}{A}\int \mathrm{d}A\, p_\omega(r) = \hat{p}_\omega(x)$$

$$\hat{f}_\omega = \frac{1}{A}\int \mathrm{d}A\, f_\omega(r)$$

These are the effective plane wave values of quantities. Taking the cross-section average of Eq. (1) and assuming a constant area combustor

$$\frac{\mathrm{d}^2 \hat{p}_\omega}{\mathrm{d}x^2} + k^2 \hat{p}_\omega + \frac{1}{A}\int_C \nabla p_\omega \cdot n \,\mathrm{d}C = -\hat{f}_\omega \qquad (2)$$

where the divergence theorem has been used.

The linearized momentum equation for the acoustic parts of p_ω and v_ω is[2]

$$i\omega v_\omega + \frac{\partial}{\partial x}(\bar{u} v_\omega) = -\frac{\nabla p_\omega}{\bar{\rho}} \qquad (3)$$

where the a subscript has been dropped. In the outward normal direction this becomes

$$\bar{\rho} i \omega v_\omega \cdot n = -\nabla p_\omega \cdot n$$

Defining the specific wall admittance as

$$\beta_w = v_\omega \cdot n \bar{\rho}\bar{c}/\hat{p}_\omega$$

and assuming β_w is uniform along and around the walls, Eq. (2) becomes

$$\frac{\mathrm{d}^2 \hat{p}_\omega}{\mathrm{d}x^2} + k^2 \hat{p}_\omega - ik\hat{p}_\omega \beta_w \frac{C}{A} = \frac{\mathrm{d}^2 \hat{p}_\omega}{\mathrm{d}x^2} + \mathrm{K}^2 \hat{p}_\omega = -\hat{f}_\omega \qquad (4)$$

The effect of β_w, if it has a positive and real part, is to introduce damping at the wall. In leading to Eq. (4) the convective term in Eq. (3) has been dropped. While it may sometimes be of the same order as the first term, what is really being done here is to introduce an effective β_w which will later be experimentally determined, but contains the wall damping effect. While the convective term which has been dropped is not necessarily a damping term, it is found in Ref. 3 that a positive, real β_w is necessary to match the observed and theoretical spectra. This corresponds to damping so that there appears no serious error in dumping the convective term into an equivalent damping term.

At the combustor head end, it is assumed the wall (even if there are air entrance slots) is acoustically hard so that $u_\omega(0,y,z) = \hat{u}_\omega(0) = 0$. Equation (3) then yields

$$\left.\frac{\mathrm{d}\hat{p}_\omega}{\mathrm{d}x}\right)_{x=0} = 0 \qquad (5)$$

At the exhaust end, an admittance condition is involved for the plane wave quantities that

$$\frac{\hat{u}_{\omega_a} + \hat{u}_{\omega_v}}{\bar{c}} + a_\omega \frac{\hat{p}_{\omega_a}}{\bar{p}} + b_\omega \Sigma_\omega = 0 \qquad (6)$$

In several limits the admittance coefficients, a_ω and b_ω, may be calculated. For example, if the combustor termination is a choked nozzle and the frequency is low[7] $a_\omega \to -(\gamma-1)M_e/2\gamma$ and $b_\omega \to -M_e/2$; if the nozzle is a pipe open to free space and the Mach number is low, the Levine-Schwinger problem[8] is the appropriate limit, whereby as frequency tends toward zero, $a_\omega \to \infty$ and $b_\omega = 0$. In general, however, a_ω and b_ω must be measured.

The admittance relation of Eq. (6) includes the vortical component of velocity. However, in the plane wave mode u_{ω_v} must be zero because the vortical velocity is dilatation free. In the higher modes it would appear as a nonzero contribution. It is possible that higher moments of the vortical velocity may contribute to the plane wave mode even though the frequencies considered are below cut-on of the higher modes. For consistency, however, one should also consider this effect for the other sources, if one were to consider it for vorticity noise. Accordingly, $\hat{u}_{\omega_v}=0$ is assumed in the following equations.

Taking the axial component of Eq. (3) and averaging over the cross section,

$$i\omega\hat{u}_{\omega_a} + \bar{u}_e \frac{d\hat{u}_{\omega_a}}{dx} = -\frac{1}{\bar{\rho}_e}\frac{d\hat{p}_{\omega_a}}{dx} \tag{7}$$

at the exit plane where $d\bar{u}/dx=0$ has been assumed. The linearized continuity equation at the exit plane is,[3] when averaged over the exit plane,

$$i\omega\hat{\rho} + \bar{u}_e \frac{d\hat{\rho}_\omega}{dx} + \bar{\rho}_e \nabla\cdot v_{\omega_a} = 0 \tag{8}$$

Next the perfect gas equation of state is introduced and, when averaged over the exit plane

$$\frac{\hat{\rho}_\omega}{\bar{\rho}_e} = \frac{1}{\gamma}\frac{\hat{p}_\omega}{\bar{p}} - \Sigma_\omega \tag{9}$$

Applying the divergence theorem to $\nabla\cdot v_{\omega_a}$, using the wall admittance condition, and combining Eqs. (7-9), there results

$$\frac{d\hat{p}_{\omega_a}}{dx} - ik\hat{p}_{\omega_a}\left[M_e + \gamma a_\omega + \frac{C}{A}\frac{M_e\beta_w}{ik}\right] = \gamma\bar{p}\left[\Sigma_\omega ik(b_\omega - M_e) - M_e^2\frac{d\Sigma_\omega}{dx}\right] \tag{10}$$

as the exit plane boundary condition. However, the energy equation, written in the form of the second law of thermodynamics, demands that the entropy fluctuations be convected with the fluid. Hence, at the exit plane Eq. (10) simplifies to

$$\frac{d\hat{p}_{\omega_a}}{dx} - ik\hat{p}_{\omega_a}\left[M_e + \gamma a_\omega + \frac{C}{A}\frac{M_e\beta_w}{ik}\right] = ik\gamma\bar{p}[b_\omega\Sigma_\omega] \tag{11}$$

The second term in brackets is generally of the order of M_e and would ordinarily be neglected if the procedure used to derive Eq. (1) were used. However, this term provides damping and prevents unbounded resonances. It is important to retain mean flow-dependent terms when they add damping, whereas in Eq. (4) the flow terms neglected shift resonant frequencies only a slight amount.

The right-hand side of Eq. (11) contains a source term at the combustor termination. The combustion noise source term occurs in the differential Eq. (4). Damping is provided by the wall impedance in Eqs. (4) and (11), by the nozzle through a_ω in Eq. (11), and by convection of acoustic energy by the mean flow in Eq. (11). Hence, all of the basic physics are included, although some approximations have been introduced.

Solution and Orders of Magnitude

Now dropping the "hat" superscript and a subscript, the solution to Eq. (4) subject to Eqs. (5) and (11) may readily be obtained. It is

$$\frac{p_\omega(x)}{\bar{p}} = A_\omega H_{Ap} + \sigma_\omega H_{\sigma p}$$

$$H_{Ap} = \frac{ik\gamma M_e}{D}\frac{\Delta T}{T_e}\left[\cos K(x-\ell) + \frac{\alpha_e}{K}\sin K(\ell - x)\right]$$

$$H_{\sigma p} = \frac{ik\gamma M_e \Delta T/T_e}{D}\left[\frac{b_\omega}{M_e}\cos Kx\right]$$

$$D = \alpha_e \cos K\ell - K\sin K\ell \quad \alpha_e = -ik\left(M_e + \gamma a_\omega + \frac{C}{A}\frac{M_e\beta_w}{ik}\right) \tag{12}$$

The two source terms, A_ω and σ_ω, are defined in the Nomenclature and are basically the Fourier transforms of the *fractional* heat release rate and exit plane entropy fluctuation. From the first of Eqs. (12) the pressure transform is linearly related to the two causal processes.

It is desirable to gain an idea of some orders of magnitude from Eqs. (12). Assume for the moment that $\sigma_\omega = 0$ so that only combustion noise is present. Furthermore, assume that a combustor is terminated by an impedance matching device so that only downstream acoustic waves are propagating (no reflections). In such a case, $a_\omega \approx -1/\gamma$ and if $M_e \ll 1$ then $\alpha_e \approx -ik$. Moreover, for low β_w, $k \approx K$. With these approximations, at $x=\ell$,

$$\frac{p_\omega(\ell)}{\bar{p}} = \frac{\gamma i M_e \Delta T/T_e}{-i\cos k\ell + \sin k\ell}A_\omega$$

Constructing a spectral density from the Fourier transform[9]

$$2\pi\frac{p_\omega p_\omega^*}{t_0\bar{p}^2} = \frac{S_p}{\bar{p}^2} = (\gamma M_e \Delta T/T_e)^2 S_A$$

The acoustic power is given by

$$P = A_e\int_{-\infty}^{\infty}\frac{S_p}{\bar{\rho}_e\bar{c}_e}d\omega = \frac{A_e\bar{p}^2}{\bar{\rho}_e\bar{c}_e}(\gamma M_e\Delta T/T_e)^2\int_{-\infty}^{\infty}S_A\,d\omega$$

$$= A_e\bar{c}_e^3\bar{\rho}_e(M_e\Delta T/T_e)^2\langle A^2\rangle$$

It is to be noted that a reference inlet velocity may be defined by $\bar{\rho}_i u_{\rm ref} = \bar{\rho}_e\bar{u}_e$ and $\bar{\rho}_i/\bar{\rho}_e = T_e/T_i$. Consequently, the grouping on the right above may be written in terms of more convenient design variables as

$$P = \langle A^2\rangle A_e u_{\rm ref}^2\bar{p}\left(\frac{\Delta T}{T_i}\right)^2\frac{\gamma}{R(T_i+\Delta T)} \tag{13}$$

A correlation formula (to be presented later in this paper) which accurately predicts combustion noise experiments closely meeting the requirements of the above assumptions, has been applied to a particular run from Ref. 4. The numbers are $u_{\rm ref}=84.1$ m/s (276 ft/s), $A_e=0.145$ m^2 (225 in.^{2}), $p=451$ KPa (65.5 psia) $\Delta T=505$ K (910°R), $T_i=908$ K (1635°R), $P=152.4$ W.

When introduced into Eq. (13), $\langle A^2\rangle = 8.9\times10^{-4}$ results. Consequently, the rms value of A is roughly 0.03. Another way of saying this is that the fluctuation level in the total heat release rate is about 3% of the mean heat release rate.

Now assume that this heat release rate fluctuation remains at the same level in an installed configuration where the combustor is terminated by a choked or nearly choked nozzle.

In such a case $b_\omega \approx -M_e/2$ for short nozzles.[7] Consequently, in looking at H_{op} and H_{Ap} in Eqs. (12), the two terms in the equation for $p_\omega/\bar{p}$ scale almost exactly the same and in fact are of the same order of magnitude if $\langle A^2 \rangle$ and $\langle \sigma^2 \rangle$ are of the same order of magnitude. This is expected to be so, since relative intensities of turbulence are high in a combustor[10] and temperature fluctuation levels are high.[3] Of course, σ is a cross-section average which will depress it in magnitude compared with its point magnitude. But it is expected that this quantity will be on the order of 1-3%. The conclusion is that the interior pressure fluctuation may be expected to be strongly influenced by both mechanisms.

This does not say anything about the noise that gets out, since 1) there are transmission properties of the combustor termination to worry about and 2) the nozzle generates additional downstream noise by the entropy mechanism.

Equation (13) gives an idea of the scaling laws to be expected in the absence of entropy noise. The only major problem is in the estimation of the scaling of $\langle A^2 \rangle$. For order of magnitude purposes, the cosKx term in the definition of A can be omitted and

$$\langle A^2 \rangle = \frac{(\int Q' \mathrm{d}V)^2}{(\int \bar{Q} \mathrm{d}V)^2}$$

Estimation of the square of stochastic variables such as Q when integrated over space yields

$$\langle A^2 \rangle \propto V V_{\mathrm{cor}} \frac{\langle Q'^2 \rangle_{\mathrm{av}}}{(\int \bar{Q} \mathrm{d}V)^2} = \frac{V_{\mathrm{cor}}}{V} \frac{\langle Q'^2 \rangle_{\mathrm{av}}}{\bar{Q}^2_{\mathrm{av}}}$$

Reference 1 assumed that the correlation volume was the entire volume of reactants produced around a single fuel nozzle. While it may be plausible that there is a certain degree of correlation within the volume served by a single fuel nozzle, there cannot be perfect correlation since there are many uncorrelated turbulent eddies within such a volume. Moreover, the turbulence size scale is probably most closely linked to a transverse combustor dimension, as in pipe flow. Consequently, $V_{\mathrm{cor}}/V \propto (A_e^{1/2}/\ell) N_f^{-a}$ seems reasonable to assume with $0<a<1$. Then if it is argued that the mean fluctuation level of Q' is proportional to the mean heat release rate, an assumption which seems reasonable in the absence of other information, $\langle A^2 \rangle \propto (A_e^{1/2}/\ell) N_f^{-a}$ results. When this is placed back into Eq. (13)

$$P \propto N_f^{-a} (A_e^{1/2}/\ell) A_e u_{\mathrm{ref}}^2 \bar{\rho} \left(\frac{\Delta T}{T_i}\right)^2 \frac{1}{T_i + \Delta T} \tag{14}$$

results. This suggests the variables against which to test and correlate the data. Moreover, it suggests the form of the correlation function.

Comparison between Experiments on Different Rigs

Often in the literature "apples and oranges" have been compared when comparing combustor noise results. The solution of Eqs. (12) clearly shows that the combustor termination (through α_e) plays a strong role in the interior pressure fluctuation. Consequently, it may be expected to play a strong role in the noise output also. Different terminations have been used on different experiments. In order to make valid comparisons between experiments, the results must be corrected to equivalent terminations.

Three cases will be considered: 1) a combustor terminated by an open unflanged end, open to the atmosphere, 2) a termination by a short, choked nozzle; and 3) a termination by either a perfect impedance matching device or, what is equivalent, an infinite extension to the pipe at the exit plane. Where necessary, it is assumed that the combustor is a right circular cylinder. Direct combustion noise is the only noise source under consideration for this demonstration and it is assumed that A_ω is unaffected by the termination for the same operating conditions (there is no reason to suspect otherwise). The exit Mach number is assumed low and $ka \ll 1$ so that in case 1 the Levine-Schwinger result must be applied that[8]

$$a_\omega = -1/\gamma ka(0.6i + 0.25ka)$$

In case 2 the nozzle is assumed short so that

$$a_\omega = -\frac{\gamma - 1}{2\gamma} M_e$$

In case 3 the Mach number is assumed low so that

$$a_\omega = -\frac{1}{\gamma}$$

For purposes of calculation β_w is assumed small so that $K \to k$. Where necessary (in case 1), $\ell/a = 7.52$ has been taken, corresponding to the rig of Ref. 1.

When only combustion noise is under consideration, the exit plane pressure along with the exit plane impedance are sufficient to calculate the power output. Using Eqs. (12) and the admittances above, and calculating the power per unit frequency interval, the results may be expressed in the form $P_{\omega\mathrm{case}\ 1}/P_{\omega\mathrm{case}\ 3}$ and $P_{\omega\mathrm{case}\ 2}/P_{\omega\mathrm{case}\ 3}$. The results for $M_e = 0.3$ are shown in Fig. 1. It is seen that the results are quite frequency sensitive, but that either of the reflecting terminations causes a substnatial drop in radiated power, compared with a nonreflecting termination. For the open end (case 1) the ¼ and ¾ wave resonances are clearly seen, whereas for the short choked nozzle case the ½ wave resonance is present. This result shows clearly that, if results from case 3 and case 1, for example, are compared experimentally, a correction of the order of 10 dB in radiated power must be applied to the results. Moreover, the correction must carefully consider the frequency content of the noise. Such a correction will be used in the next section.

III. Correlation of Past Combustor Rig Tests

In Refs. 1, 4, and 10-12, there is a wealth of data regarding combustion noise from combustor rig tests. Virtually all relevant parameters have been varied, tests at simulated engine conditions have been conducted, and different combustor types were used. Comparison of correlations produced for the data, however, shows that the correlation of one manufacturer does not cover that of another manufacturer. It is the purpose here to see if a unified correlation can be developed which will adequately cover the data ranges of the cited references.

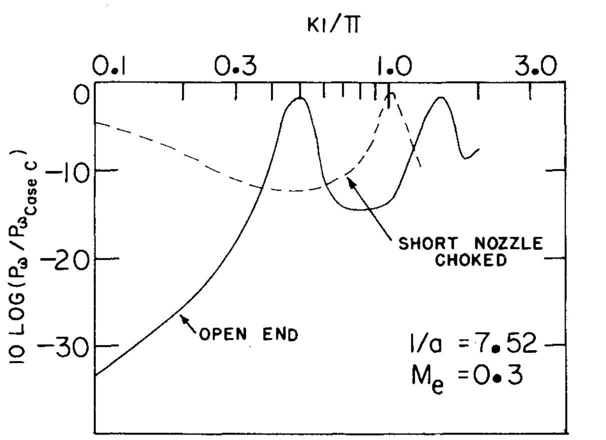

Fig. 1 Comparison of sound radiated per unit frequency interval with that radiated from reflection free termination.

Table 1 Regression analysis results and comparisons with theory

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Experiment	0.91	1.9	3.4	−2.5	1.3	−0.78	1.0	1.0
Theory	N/A	1	2	−2 to −3	2	0 to −1	1	1

The work here will concentrate on sound power only. The frequency content of the cited references poses one of the great unsolved problems in combustion noise.[13] In Ref. 1 a theory is presented for the frequency content which appears to recover the correct trends for the combustors of that work, but it works for no other manufacturers'. Moreover, the results of Ref. 10 are not explained by that theory. About all that can really be said is that combustion noise has been observed in the past to be basically low frequency in nature (≤ 1000 Hz). Of course, it must be borne in mind that the frequency content depends upon the acoustics of the containing hardware. Since the combustor rigs have been different, it is perhaps not surprising that there is confusion concerning the observed frequency content. In any event, the issue will be avoided here. Only sound power will be considered.

In the rigs of Refs. 1 and 10 the six different combustors were exhausting directly (with some diffusion before exit) to the atmosphere. The acoustic configuration was essentially that of an open-ended organ pipe and resonances are clearly seen in the data at the 1/4, 3/4, 5/4, etc., wave resonance points. On the other hand, the work of Refs. 4, 11, and 12 was carried out in two different rigs which were assumed reflection free at the combustor termination. The data reduction procedure assumed that waves emanating from the combustor were not reflected from downstream impedance mismatches. In viewing these rigs, it is the judgment here that such an assumption was probably justified, at least to a reasonable approximation. Consequently, the data of the various rigs cannot be directly compared unless corrections are applied to bring the data to the same basis.

The basis chosen here is to correct all acoustic power measurements to the "equivalent reflection free power," P_{eq}. Thus, the work of Refs. 4, 11, and 12 will be accepted as is and corrections will be applied to the data of Refs. 1 and 10. This correction will be simply done. It is noted in Ref. 1 that the ¼ wave resonance occurs near 125 Hz whereas the main combustion noise spectrum usually peaked near 500 Hz. Since on Fig. 1 the ¼ wave resonance occurs at $k\ell/\pi = 0.5$ for the "open end" curve, the 500 Hz point corresponds to $k\ell/\pi = 2$ where a 7 dB correction is implicit in Fig. 1. This 7 dB will be added uniformly to the sound power results of all tests from Ref. 1. For accurate results the correction should be applied to the experimental spectra and then a spectral summation should be applied. However, in view of the results below, the current procedure appears to yield an adequate approximation.

In Ref. 10 most of the noise was centered about the ¼ wave resonance point. Reference to Fig. 1 shows that a correction of 1 dB is reasonable and this will be used.

The next thing to notice is that no entropy noise is expected in the cited rig tests. As discussed above, there is no pressure gradient imposed upon the exhaust and consequently, no entropy noise ($b_\omega = 0$). Consequently, there is high confidence that a form of the correlation law suggested by Eq. (14) will satisfactorily correlate the data.

The form chosen is

$$P_{eq} = a_1 p^{a_2} u_{\text{ref}}{}^{a_3} T_i{}^{a_4} F^{a_5} N_f{}^{a_6} A_e{}^{a_7} (A_e^{1/2}/\ell)^{a_8} \qquad (15)$$

This form is close to the form of Eq. (14) and is suitable for linear multiple regression analysis after taking the logarithm

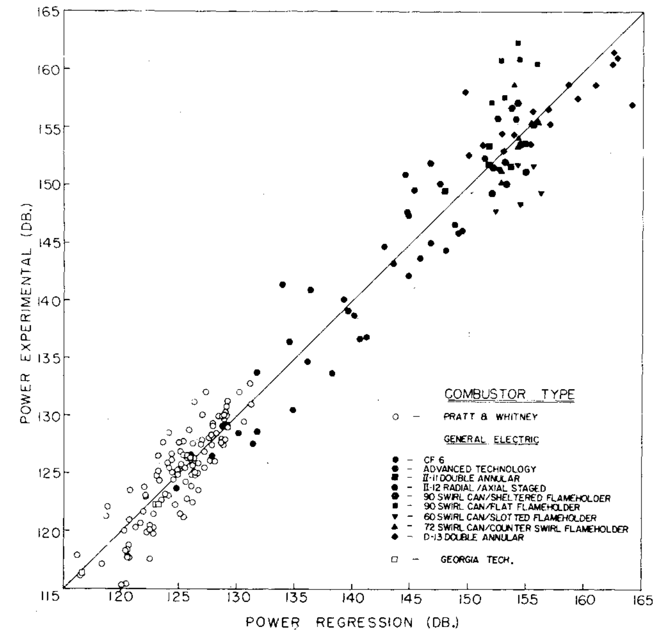

Fig. 2 Comparison of regression fit with experimental acoustic power.

of Eq. (15).‡ From the cited references, 202 data points were available from 15 different combustors. Table 1 compares the results of the regression analysis with the theoretically expected values.

The standard deviation of the correlation is ±3 dB and a comparison of the regression formula with the data is shown in Fig. 2. The individual General Electric combustors are pointed out in this figure because it was with these combustors that the largest scatter occurred. The correlation is considered highly interesting for several reasons: 1) there is a rational theoretical basis which generally holds up, 2) the single correlation formula holds over an extreme range of acoustic power (10^6), and 3) the Ref. 1 conclusion of a fuel nozzle effect holds up when all combustors are considered. This fuel nozzle effect also held up within the General Electric group of combustors.

A close look at the General Electric designs revealed no design parameter which, if included in the correlation, would reduce the scatter. Consequently, the ±3 dB is accepted as primarily due to experimental error in the variety of rigs employed, in data analysis methods, and in, perhaps, some differences in termination behavior not accounted for here.

It is to be noted that if the results of Ref. 1 are not changed by the 7 dB correction, the a_7 exponent comes out as 2.0 rather than 1.0. This is not acceptable from the theoretical viewpoint and is viewed as confirmation of the correction procedure. However, it should be borne in mind that the termination behavior assumed was that corresponding to the no-flow case and that significant differences may occur with flow.[13] As the flow increases the jet pipe becomes more and more anechoic, which would cause an upward lift of the "open end" curve of Fig. 1. This would cause a correction

‡Note: The units are power in W, pressure in psia, velocity in ft/s, inlet temperature in °R, and exit area in in². All other quantities are dimensionless.

somewhat less than the 7 dB which was actually applied; correspondingly, the a_7 exponent is somewhat less than 2. However, in view of the uncertainties involved in this hot flowing jet and the agreement with theory, no further correction will be made.

The primary deviation between theory and experiment lies in the a_2 and a_3 exponents. There is an interesting interpretation for this. The deviation from theory is by a factor of

$$P^{0.9} u_{\text{ref}}{}^{1.4} \approx (pu_{\text{ref}}^2)^{0.9} \propto (\rho u_{\text{ref}}^2)^{0.9}$$

That is, the deviation looks closely related to the dynamic head, a term which often enters in aerodynamic noise computations. Moreover, aerodynamic noise usually scales with a high exponent on velocity. It is conjectured here that since no experiment can be entirely free from aerodynamic noise contamination of the combustion noise results, the high a_2 and a_3 exponents may be false. It is possible that the theory is more accurate as to combustion noise scaling laws than to the experiments.

It is emphasized that this is a combustor rig correlation for equivalent power in a reflection-free situation and in the absence of entropy noise. In an engine installation where the combustor is terminated by nozzle guide vanes, the turbine assembly, and exhaust duct, several things change which will affect the acoustic power output. First, the nozzle guide vanes usually operate at a choked or nearly choked condition. This necessitates corrections by a form of Fig. 1 and also brings in the possibility of entropy noise. Second, more noise due to entropy can be created in the turbine, and this is combustor-related. Finally, there are attenuation processes in the turbine and nozzle which must be accounted for.

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